MATH REVISION

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Math REVESION

Qena Student Club

Math L.O.7

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Concepts

- 1. Quadratic Function
- 2. First and second differences
- 3. Completing the square
- 4. Complex numbers
- 5. Parabola
- 6. Focus
- 7. Argand diagram
- 8. related roots

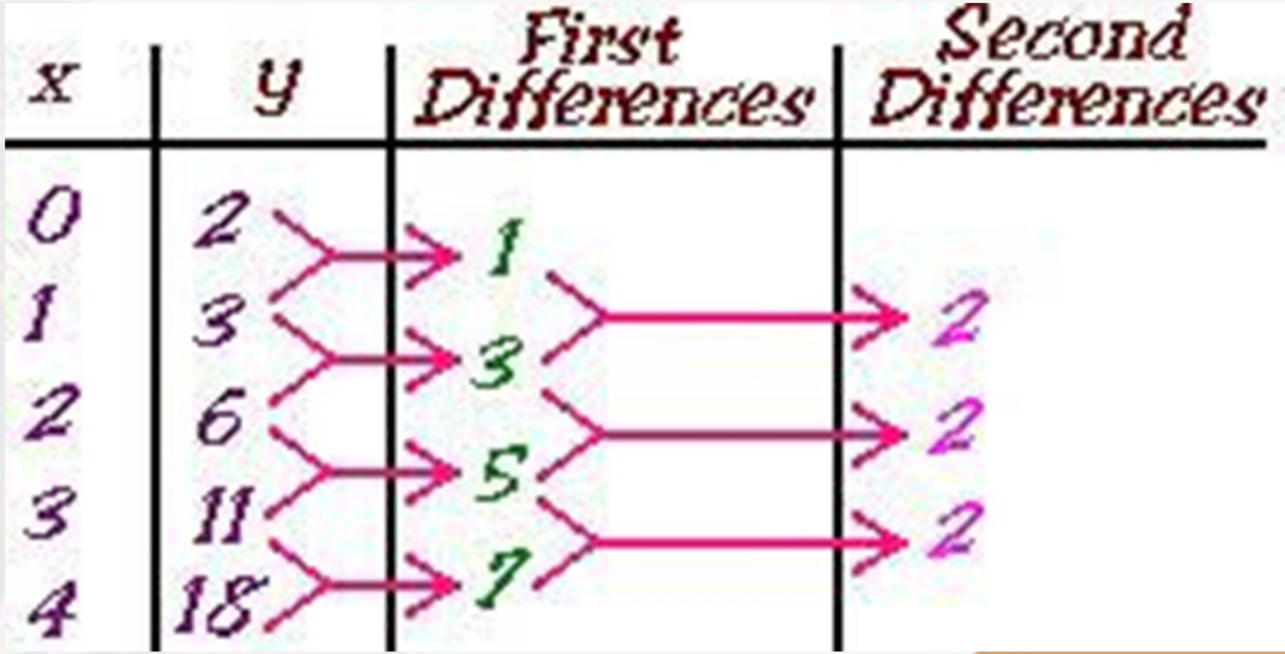
Quadratic function

- A quadratic function is a polynomial function of
- degree 2 . So a
- quadratic function is a function of f x = ax^2 + bx + c , a≠0
- The solution of a quadratic equation
- Ax^2 + bx + c is given by
- $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
- the expression b^2 – 4ac is called the discriminant

- ✓ $D>0$ then the roots are real and distinct
- ✓ $D=0$ then the roots are real and equal
- ✓ $D<0$ then the roots are imaginary and distinct
- ✓ $D>0$
- ✓ $D>0$ and a perfect square, then the roots are rational
- ✓ $a=1$, and not a perfect square, the roots are irrational
- **b, c ∈Q, and D is a perfect square, then both the roots are integers**
- **A+b+c=0, then 1 is one root and the other root will be (ca)**
- **A+b+c=0,** −1**is one root and the other root is** − ca
- **the equation (ax^2 +bx + c)has real roots α**
- **and β, we write ax^2 +bx + c** =ax – a (x-B)
- is a polynomial function of second degree that has almost 2 solution
- solution=zeroes=roots=X-intercept
- they are mean the values of x to make f(x)=0
- When a<0 the function will be downward
- When a>0 the function will be upward

First and second differences

If first differences are equals the function will be liner but if the second which are equals the function will be quadratic function



Complex number

- When you try to solve a quadratic function, you get three
- possibilities, one of them when $\boldsymbol{D}<\mathbf{0}$. In this one the
- solutions are imaginary. You must know complex numbers
- to solve these equations.

Two complex numbers are equal if and only if the two real parts are equal and the two imaginary parts are equal.

if: $a + bi = c + di$ then: $a = c$ and $b = d$ and vice versa.

Example:

Find in the simplest form the result of each of the following:

A $(7 - 4i) + (2 + i)$

Solution

The expression $(7 - 4i) + (2 + i)$

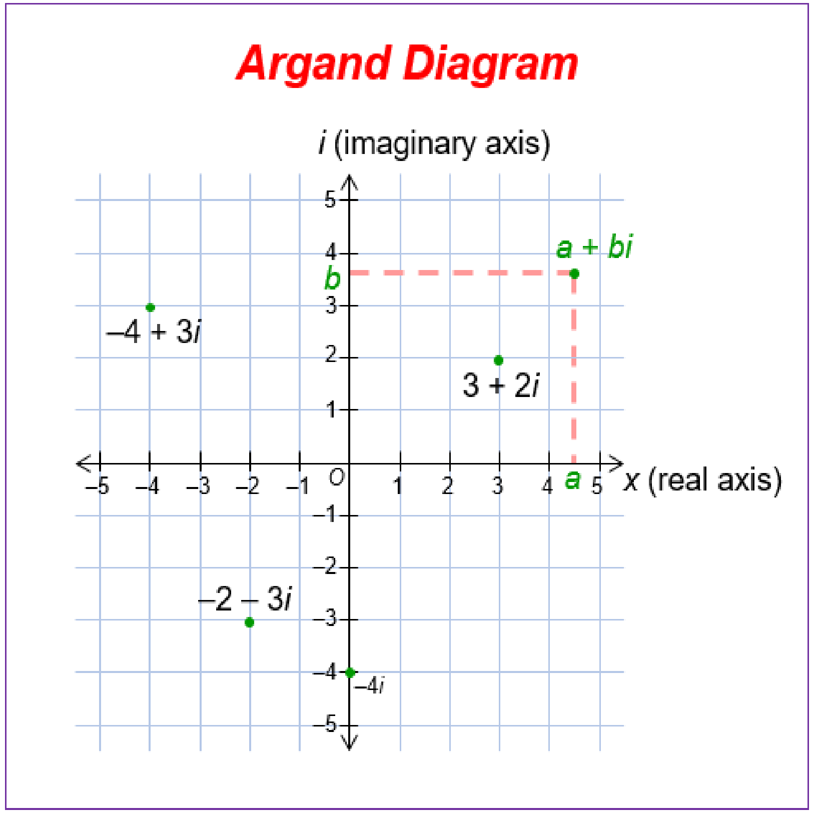
$= (7 + 2) + (-4 + 1)\,i$ Commutative and associative properties

$= 9 - 3\,i$ Simplify

Argand diagram

What is the point on an Argand diagram?

Any complex number z can be represented by a point on an Argand diagram. We can join this point to the origin with a line segment. The length of the line segment is called the modulus of the complex number and is denoted |z|



- **Example 1**

Plot the following complex numbers on an Argand diagram.

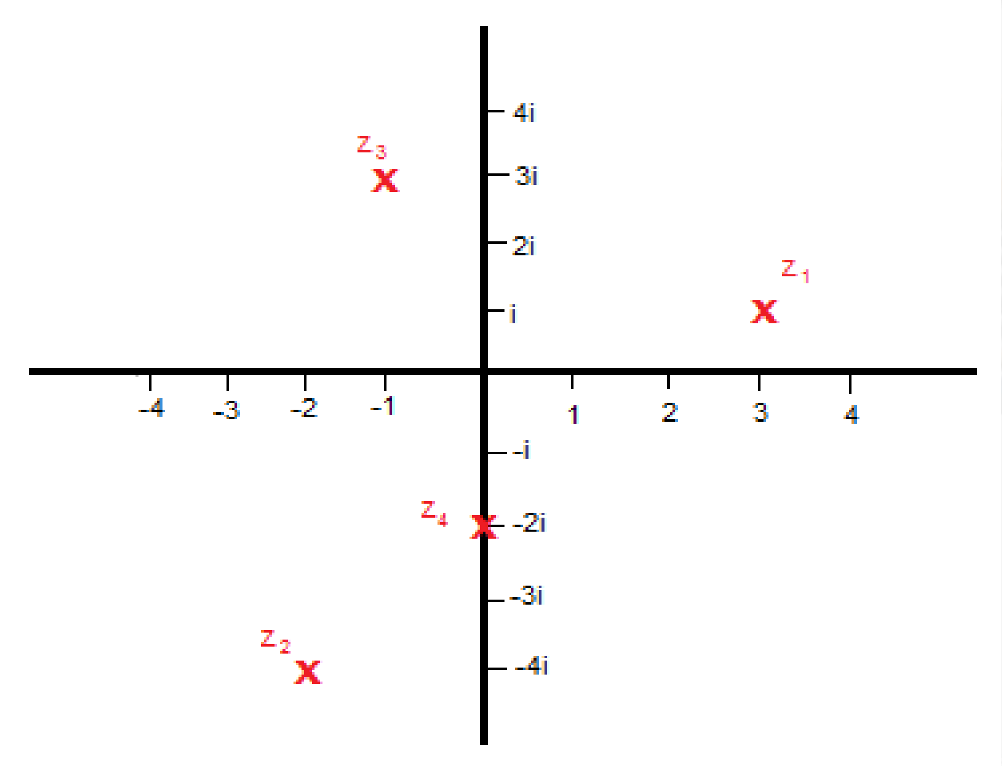
z1=3+I

z3=−1+3i

z4=−2i

z2=−2−4i

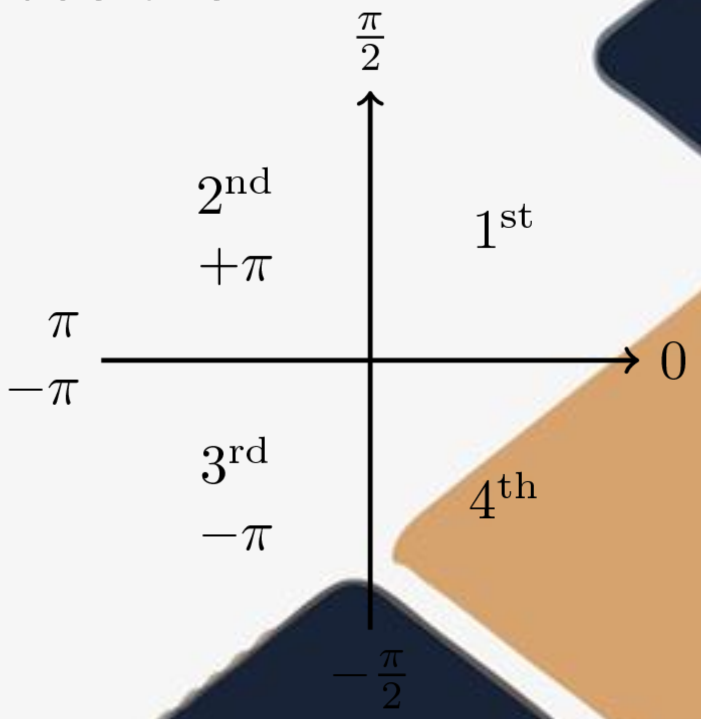
- **Solution**



- The modulus of a complex number z=a+bi is

|z|=√a2+b

When calculating the argument of a complex number, there is a choice to be made between taking values in the range [−π,π] or the range [0,π] Both are equivalent and equally valid. On this page we willuse the convention −π<θ<π.



The 'naive' way of calculating the angle to a point (a,b) is to use arctan(b/a)arctan but, since arctan only takes values in the range [−π/2,π/2]this will give the wrong result for coordinates with negative x-component.

You can fix this by adding or subtracting π,

depending on which quadrant of the Argand diagram the point lies in.

Completing the square

How do you solve quadratics by completing the square?.

Solve quadratic equations by inspection, taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

Perfect Square Trinomials

Examples

- $X2 + 6x + 9$
- $X2 - 10x + 25$
- $X2 + 12x + 36$

Creating a Perfect Square Trinomial : _

- In the following perfect square trinomial, the constant term is missing.

X2 + 14x + ____

- Find the constant term by squaring half the coefficient of the linear term. (b/2)2
- (14/2)2

X2 + 14x + 49

Create perfect square trinomials:

- $x^2 + 20x +$ ____
- $x^2 - 4x +$ ____
- $x^2 + 5x +$ ____

Ans:

1-100

2-4

3-25/4

Solving Quadratic Equations by Completing the Square:

Solve the following equation by completing the square:

$$x^2 + 8x - 20 = 0$$

Step 1: Move quadratic term, and linear term to left side of the equation

$$x^2 + 8x = 20$$

Step 2: Find the term that completes the square on the left side of the equation.

Add that term to both sides.

$$x^2 + 8x + \square = 20 + \square$$

$$\frac{1}{2} \bullet (8) = 4 \text{ then square it, } 4^2 = 16$$

$$x^2 + 8x + 16 = 20 + 16$$

Step 3: Factor the perfect square trinomial on the left side of the equation. Simplify the right side of the equation.

$$x^2 + 8x + 16 = 20 + 16$$

$$(x-4)(x-4) = 36$$

$$(x-4)^2 = 36$$

Step 4: Take the square root of each side

$$\sqrt{(x+4)^2} = \sqrt{36}$$

$$(x+4) = \pm 6$$

Step 5: Set up the two possibilities and solve

$$x = -4 \pm 6$$

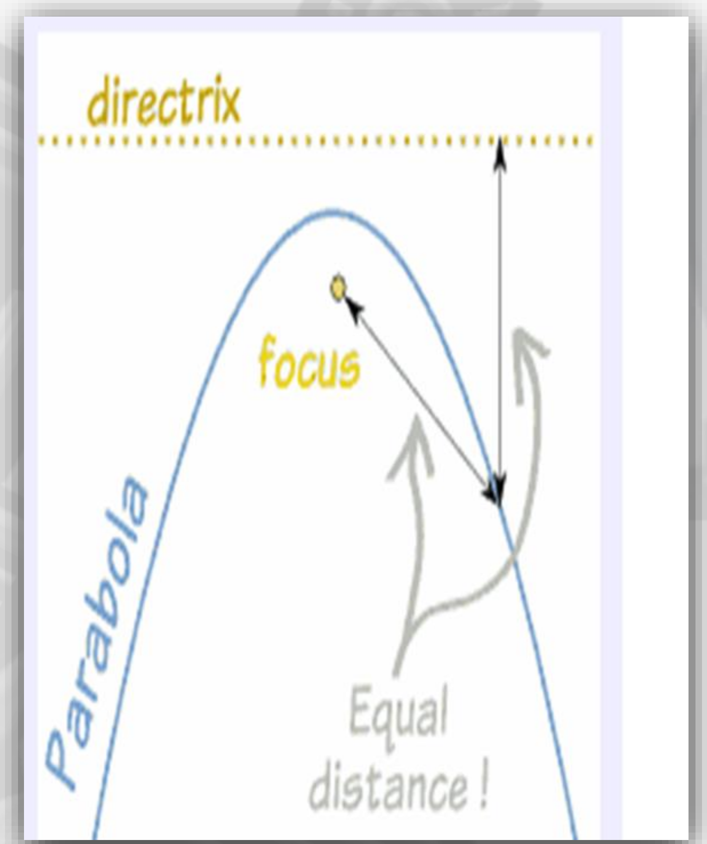
$$x = -4 - 6 \text{ and } x = -4 + 6$$

$$x = -10 \quad \text{and} \quad x = 2$$

The Parabola

The parabola is the locus of all points in a plane that are the same distance from a line in the plane, the directrix, as from a fixed point in the plane, the focus.

Point Focus = Point Directrix
PF = PD



The parabola has one axis of symmetry, which intersects the parabola at its vertex.

The distance from the directrix to the vertex is also | *p* |.

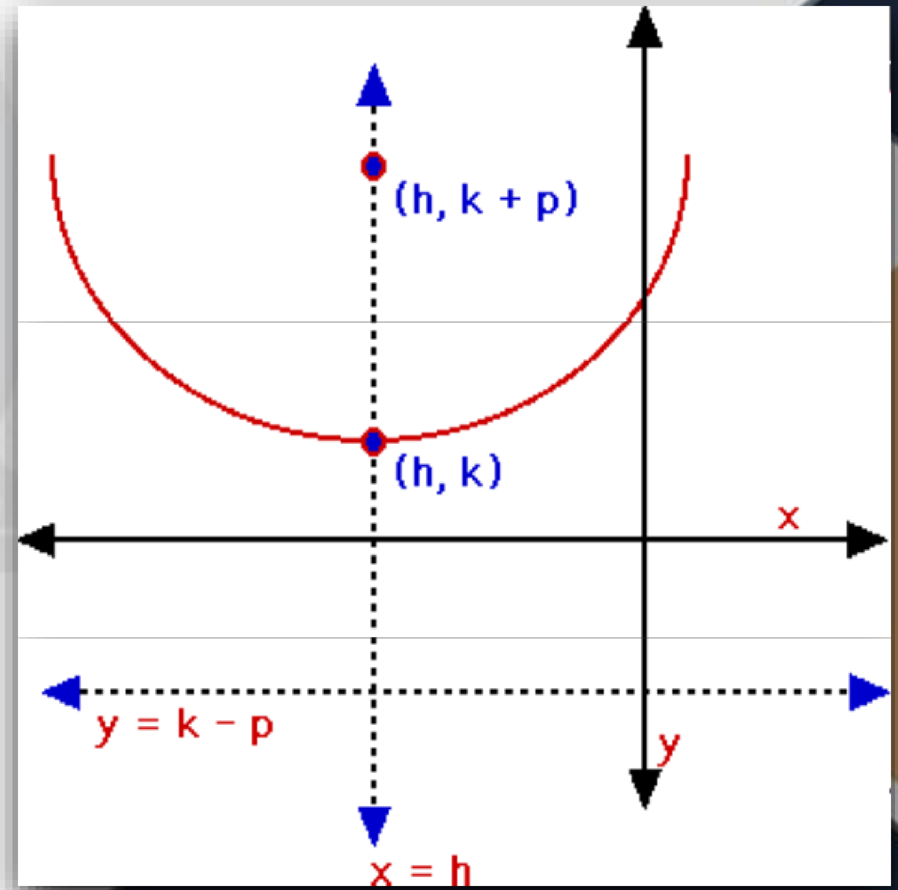
The distance from the vertex to the focus is | *p* |.

The Standard Form of the Equation with Vertex (*h, k*)

- For a parabola with the axis of symmetry parallel to the *y*-axis and vertex at (*h, k*), the standard form is …

$$(x - h)^2 = 4p(y - k)$$

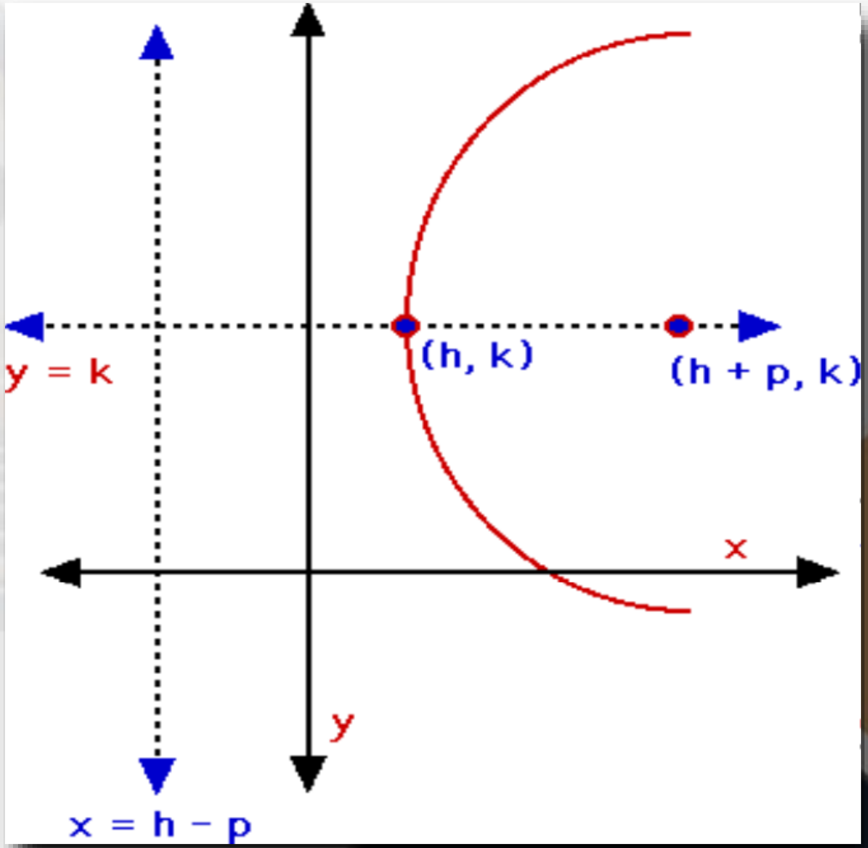
- The equation of the axis of symmetry is $x = h$.
- The coordinates of the focus are $(h, k + p)$.
- The equation of the directrix is $y = k - p$.
- When p is positive, the parabola opens upward.
- When p is negative, the parabola opens downward.



- For a parabola with an axis of symmetry parallel to the x-axis and a vertex at (h, k), the standard form is:

$$(y - k)^2 = 4p(x - h)$$

- The coordinates of the focus are $(h + p, k)$.
- The equation of the directrix is $x = h - p$.
- When p is positive, the parabola opens to the right.
- When p is negative, the
- parabola
  opens to the left.



Finding the Equations of Parabolas

Write the equation of the parabola with a focus at (3, 5) and the directrix at $x = 9$, in standard form and general form

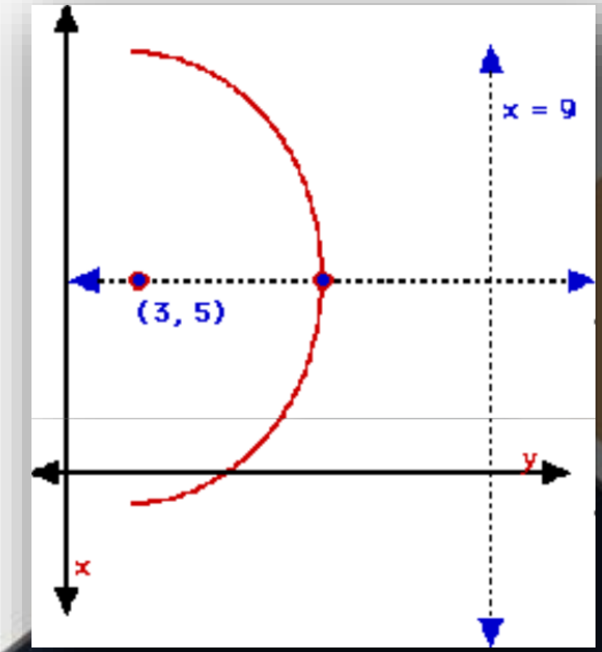
The distance from the focus to the directrix is 6 units, therefore, $2p = -6$, $p = -3$. Thus, the vertex is (6, 5).

The axis of symmetry is parallel to the *x*-axis:

$(y - k)^2 = 4p(x - h)$ $h = 6$ and $k = 5$

$(y - 5)^2 = 4(-3)(x - 6)$

$(y - 5)^2 = -12(x - 6)$ Standard form



Finding the Equations of Parabolas

Find the equation of the parabola that has a minimum at (-2, 6) and passes through the point (2, 8).

The axis of symmetry is parallel to the *y*-axis.

The vertex is (-2, 6), therefore, $h = -2$ and $k = 6$.

$$(x - h)^2 = 4p(y - k) \qquad x = 2 \text{ and } y = 8$$

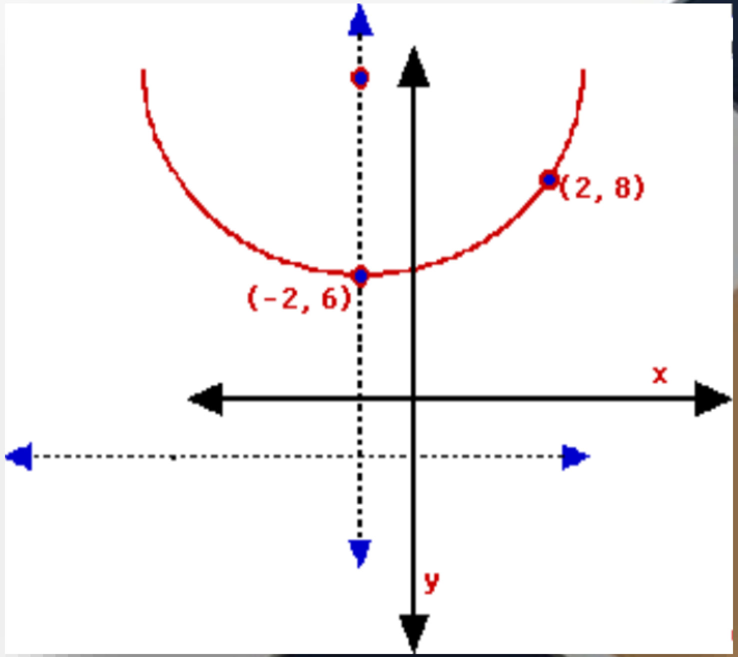
$$(2 - (-2))^2 = 4p(8 - 6)$$

$$16 = \qquad 2 = p$$

$$(x - h)^2 = 4p(y - k$$

$$(x - (-2))^2 = 4(2)(y - 6)$$

$$(x + 2)^2 = 8(y - 6)$$ Standard form



Related Roots

- Note: The discriminant: It tell us how many roots the equation has = b2-4ac
- If b2 -4ac > 0 Two different and real root
- B2 -4ac = 0 Two equal real roots
- B2 - 4ac < 0 Two imaginary roots
- Note: A2+B2= (A+B)2-2AB
- Note: (A-b)2= (A+B)2-4AB

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Concepts

- Exponential function
- Logarithmic function
- Growth
- Decay

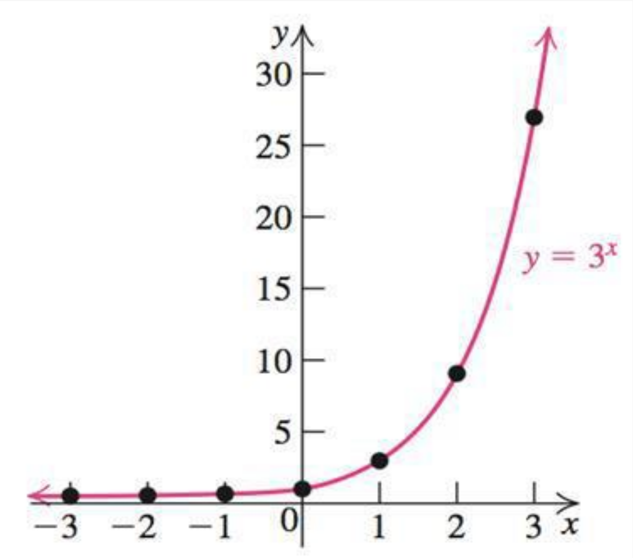
Exponential functions

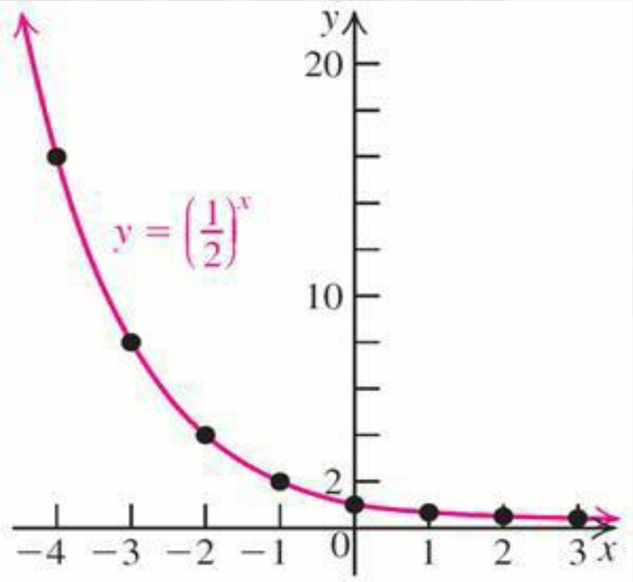
The exponential function with base a is defined by

- $f(x) = a^x$
- where $a > 0$, $a \neq 1$, and x is a real number.
- If the base were a negative number, the value of the function would be a complex number for some values of x.
- is defined such that $a \neq 1$because $f(x) = 1^x = 1$ is a constant function

Properties of exponential functions of the form $\boldsymbol{f x = a \wedge x}$,

- 1.The function is a one-to-one function as the domain of the function is $(-\infty,\infty)$and the range of the function is $(0,\infty)$.
- 2.The graph of f is a smooth, continuous curve with a y-intercept of (0,1), and the graph passes through $(1,a)$.
- 3.The graph of $f(x)=a^{\wedge}x$has no x-intercepts, so it never crosses the x-axis. No value of x will cause $f(x)=a^{\wedge}x$to equal 0.
- 4.The x-axis is a horizontal asymptote for every exponential function of the form $f(x)=x^{\wedge}a$





For a>1, Exponential Growth

f is an increasing function, so the graph rises to the right.

As x→∞, y→∞

As x→−∞, y→0

For 0<a<1, -Exponential Decay

f is a decreasing function, so the graph falls to the right.

As x→−∞, y→∞

Natural Exponential Function

-The irrational number $e$is useful in many applications that involve growth or decay.

-The letter e represents the number that (1+1/n)^n approaches as n increases without bound.

-The value of $e$accurate to eight decimal places is 2.71828183.

Natural exponential Function

-For all real numbers x, the function defined by

$f(x)=e^x$

Logarithmic Functions

• If $x>0$ and b is a positive constant ($b\neq1$), then
$y=\log bX$ if and only if $b^y=x$
The notation is read "the logarithm (or log) base b of x." The function defined by $f(x)=\log bX$ is a logarithmic function with base b. This function is the inverse of the exponential function $g(x)=b^x$

Composition of Logarithmic and Exponential Functions

Let $g(x)=b^x$ and $f(x)=\log bX$ ($x>0,b>0,b\neq1$). Then
$g(f(x))=b^{\log bX}=x$ and $f(g)x))=\log bB^x=x$

Notes:

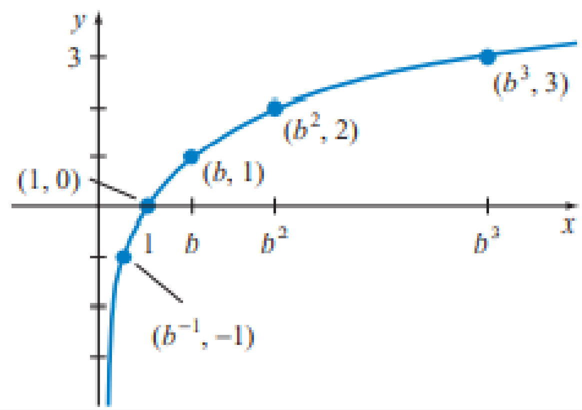
The exponential form of $y=\log bX$ is $by=x$.

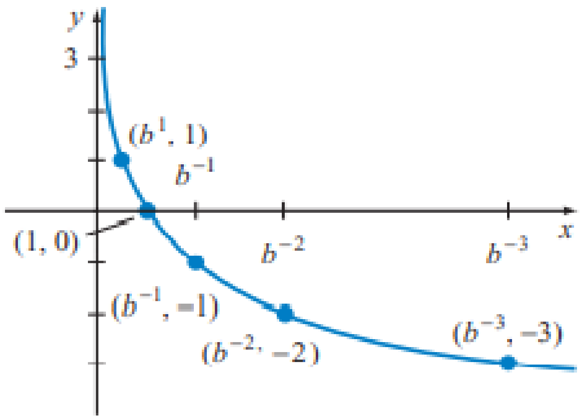
The logarithmic form of $by=x$ is $y=\log bX$.

Properties of f(x)=log bx

1.The domain of the function is $(0,\infty)$and the range of the function is $(-\infty,\infty)$.

- 2.The graph of $f$has an x-intercept of (1,0)and passes through $(b,1)$.
- 3. If $\boldsymbol{b>0}$, f is an increasing function and its graph is asymptotic to the
negative y-axis.
- $x \to \infty$, $f(x) \to \infty$
- $x \to 0$, $f(x) \to -\infty$
- 4. If $\boldsymbol{0<b<1}$, $f$is an decreasing function and its graph is asymptotic to the
positive y-axis.
- $x \to \infty$, $f(x) \to -\infty$
- $x \to 0$, $f(x) \to \infty$





Percent Increase and Decrease

You can model growth or decay by a constant percent increase or percent decrease with the formula:

$$A(t) = a\,(1 \pm r)t$$

- Initial Amount: a
- Number of Time Periods: t
- Final Amount: $A(t)$
- Rate of Increase: r

- 1+r is growth factor
- 1-r is decay factor

Growth & Decay

Growth : is when data rises over a period of time, creating an upwards trending curve on a graph. In mathematics

Decay : Is process in which a quantity decreases over time, with the rate of decrease becoming proportionally smaller as the quantity gets smaller.

Exponential Growth And Decay

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Exponential Growth	Exponential Decay
$f(x) = a(1 + r)^t$	$f(x) = a(1 - r)^t$

r - rate of growth
t - time steps

Example 1: What is the amount received from the investment fund after 2 years, if $.100,000 were invested at the compounding rate of 5% per every quarter?

Solution:

- The invested principal is a = \$100,000, the rate of compounding growth is r = 5% = 0.05 per quarter.
- The time period is 2 years, and there are 4 quarters in a year, and we have t = 8.
- Applying the concepts of exponential growth and decay we have the following expressions for exponential growth.
- $f(x) = a(1 + r)^t$
- $f(x) = 100{,}000(1 + 0.05)^8$
- $f(x) = 1{,}00{,}000(1.05)^8 = 100{,}000 \times 1.47745544 = 147745.44$
- Therefore an amount of \$1,47, 746 is received after a period of 2 years.

Example 2: The radioactive material of thorium decays at the rate of 8% per minute. What part of 10 grams of thorium would be remaining after 5 minutes?

Solution:

- The given initial quantity of thorium is a = 10grams, the rate of decay per minute is r = 8% = 8/100 = 0.08, and the time steps t = 5.
- Here we can apply the concepts of exponential growth and decay, and the exponential decay formula for the decay of thorium is as follows.
- $f(x) = a(1 - r)^t$
- $^f(x) = 10(1 - 0.08)^5 = 10(0.92)^5 = 6.5908$
- Therefore a quantity of 6.6 grams of thorium remains after 5 minutes.

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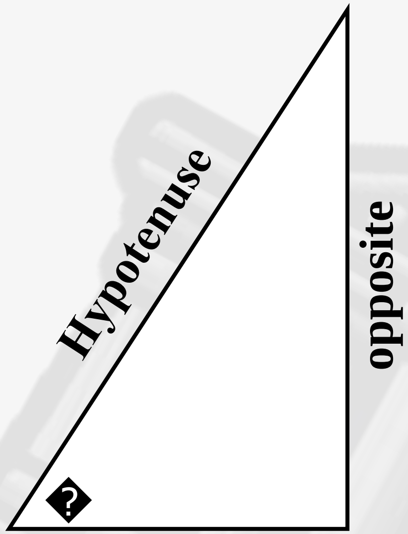
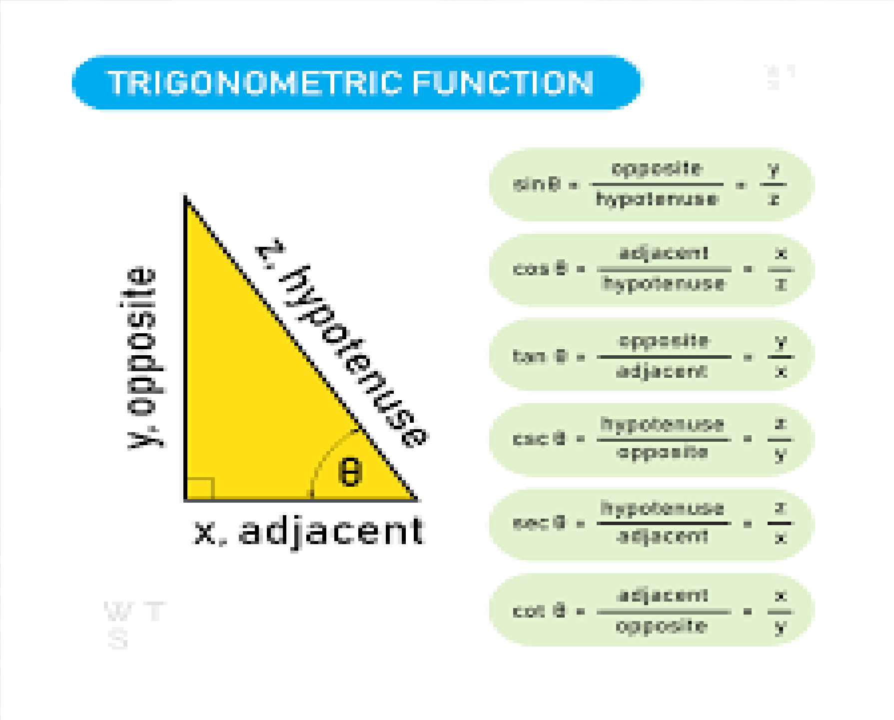
Trigonometric Functions

- ➢ The Main Identity : ***Sin*2** x + ***Cos*2** x = 1
- From this equations you can extract other equations

Trigonometric functions



Sin = opposite / hypotenuse, csc =1/sin
Cos = adjacent / hypotenuse, sec = 1/cos
Tan = opposite / adjacent, cot = 1/tan



Trigonometric Identities

Reciprocal Identities

$$\csc\theta = \frac{1}{\sin\theta}$$
$$\sec\theta = \frac{1}{\cos\theta}$$
$$\cot\theta = \frac{1}{\tan\theta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$
$$\sec^2\theta = 1 + \tan^2\theta$$
$$\csc^2\theta = 1 + \cot^2\theta$$

Half-Angle Formulas

$$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{2}}$$
$$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos\theta}{2}}$$
$$\tan\left(\frac{\theta}{2}\right) = \frac{1-\cos\theta}{\sin\theta}$$

Product to Sum Formulas

$$\sin x \sin y = \frac{1}{2}[\cos(x-y) - \cos(x+y)]$$
$$\cos x \cos y = \frac{1}{2}[\cos(x-y) + \cos(x+y)]$$
$$\sin x \cos y = \frac{1}{2}[\sin(x+y) + \sin(x-y)]$$
$$\cos x \sin y = \frac{1}{2}[\sin(x+y) - \sin(x-y)]$$

Sum to Product Formulas

$$\sin x + \sin y = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$
$$\sin x - \sin y = 2\sin\left(\frac{x-y}{2}\right)\cos\left(\frac{x+y}{2}\right)$$
$$\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$
$$\cos x - \cos y = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

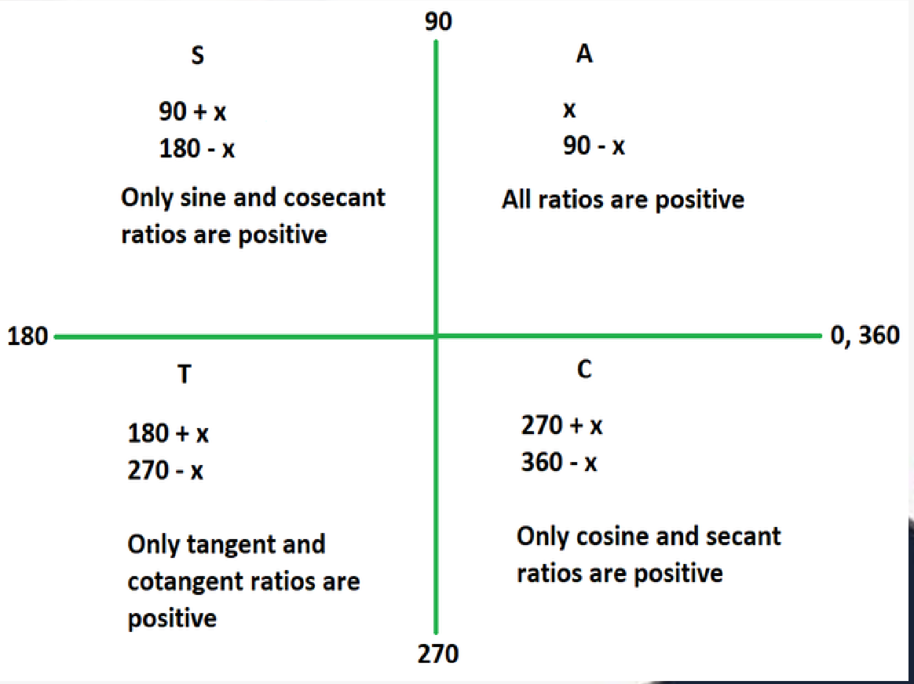
Coordinates

• As shown as in figure 1: the vertical line is called, Y-axis And the horizontal line is called X-axis

➢We use the coordinates to represents data

➢The first coordinate is divided to four parts are: the first coordinate, second coordinate, third coordinate and fourth coordinate

• the ordered pair in first coordinate is (+,+)
• the ordered pair in second coordinate is (-,+)
• the ordered pair in third coordinate is (-,-)
• the ordered pair in fourth coordinate is (+,-)



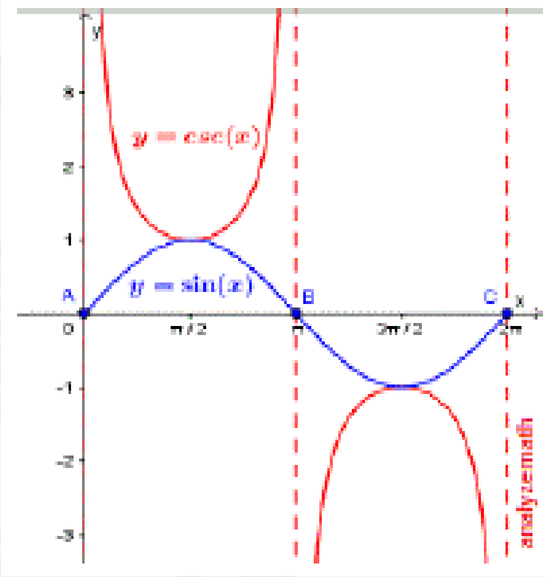
Properties of the Sine Function $y = \sin x$

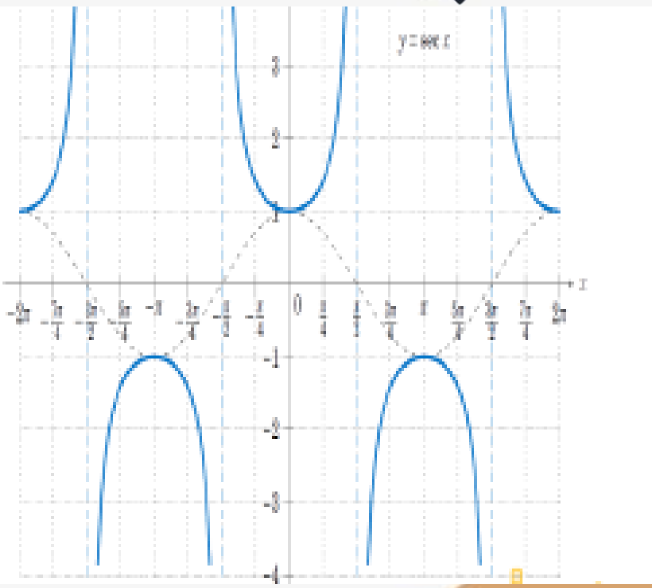
1. The domain is the set of all real numbers.
2. The range consists of all real numbers from -1 to 1, inclusive.
3. The sine function is an odd function, as the symmetry of the graph with respect to the origin indicates.
4. The sine function is periodic, with period 2π.
5. The x-intercepts are $\ldots, -2\pi, -\pi, 0, \pi, 2\pi, 3\pi, \ldots$; the y-intercept is 0.
6. The maximum value is 1 and occurs at $x = \ldots, -\frac{3\pi}{2}, \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \ldots$; the minimum value is -1 and occurs at $x = \ldots, -\frac{\pi}{2}, \frac{3\pi}{2}, \frac{7\pi}{2}, \frac{11\pi}{2}, \ldots$.

Graphs of sec, csc

- You can get these graphs from the sin and cos because there are flipped of there and theses line also asymptotes

- That is mean, these line or curved don t touch the verticals line because on these line sin X = 0 and csc is flipped of sin and you can't divide by zero the same thing in cos and sec





Properties of cosine function

1. The domain is the set of all real numbers.
2. The range consists of all real numbers from -1 to 1, inclusive.
3. The cosine function is an even function, as the symmetry of the graph with respect to the y-axis indicates.
4. The cosine function is periodic, with period 2π.
5. The x-intercepts are $\ldots, -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$; the y-intercept is 1.
6. The maximum value is 1 and occurs at $x = \ldots, -2\pi, 0, 2\pi, 4\pi, 6\pi, \ldots$; the minimum value is -1 and occurs at $x = \ldots, -\pi, \pi, 3\pi, 5\pi, \ldots$.

Inverse sine , cosine , and tangent function

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properties of a one-to-one function f and its inverse function f^{-1}

1. $f^{-1}(f(x)) = x$ for every x in the domain of f and $f(f^{-1}(x)) = x$ for every x in the domain of f^{-1}.

2. Domain of f = range of f^{-1} and range of f = domain of f^{-1}.

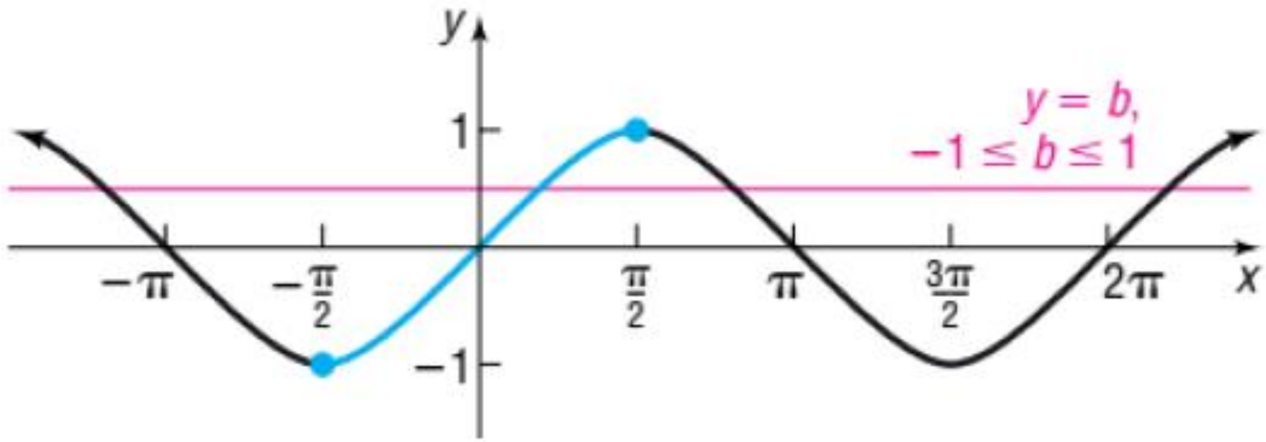
3. The graph of f and the graph of f^{-1} are reflections of one another about the line $y = x$.

Inverse sine function

•

In Figure 1, we show the graph of $y = \sin x$. Because every horizontal line $y = b$, where b is between -1 and 1, inclusive, intersects the graph of $y = \sin x$ infinitely many times, it follows from the horizontal-line test that the function $y = \sin x$ is not one-to-one.

Figure 1
$y = \sin x, -\infty < x < \infty, -1 \le y \le 1$



Domain and Range of Trigonometric Functions

Domain and Range of Trigonometric Functions are the values in which the function is defined and the output for the domain respectively. Trigonometric Ratios describe relationships between angles and the sides- of right triangles. A Function defined in terms of Trigonometric Ratios is called a Trigonometric Function. There are a total of six Trigonometric Functions. They play a significant role in various branches of mathematics and science, particularly in calculus and geometry. Trigonometric functions are essential in mathematics and have widespread applications in various fields.

What is Domain and Range?

Domain of a function is the set of input values (angles, in this case) for which the function is defined and produces a valid output. Trigonometric functions, such as sine, cosine, and tangent, have a domain that includes all real numbers. However, their range varies. **Range** of a function is the set of all possible output values. To calculate trigonometric values, you use the ratios. For example, $\sin(\theta)$ = opposite/hypotenuse, $\cos(\theta)$ = adjacent/hypotenuse, and $\tan(\theta)$ = opposite/adjacent.

Example:

Common trigonometric values include sin(30°) = 1/2, cos(45°) = √2/2, and tan(60°) = √3.

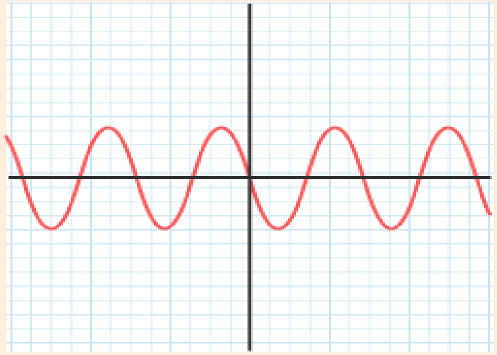
Sine and cosine functions have a range of [-1, 1], representing the amplitude of oscillation.

Tangent has a domain restriction excluding odd multiples of π/2 (-π/2, π/2, 3π/2, etc.), but its range is all real numbers.

Trigonometric Function	Domain	Range
sin(θ)	R	[-1, 1]
cos(θ)	R	[-1, 1]
tan(θ)	R excluding odd multiples of π/2	R
cot(θ)	R excluding multiples of π	R
sec(θ)	R excluding values where cos(x) = 0	R-[-1, 1]
cosec(θ)	R excluding multiples of π	R-[-1, 1]

Domain and Range of Trigonometric Functions Using Graph

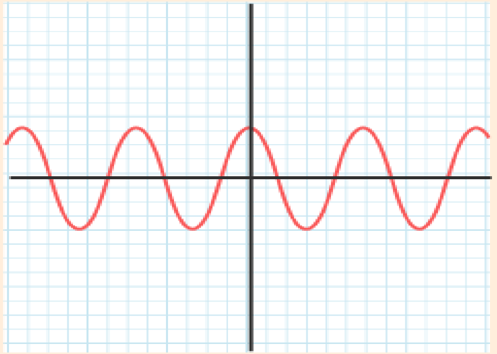
Sin θ



Domain = $(-\infty, +\infty)$

Range = $(-1, +1)$

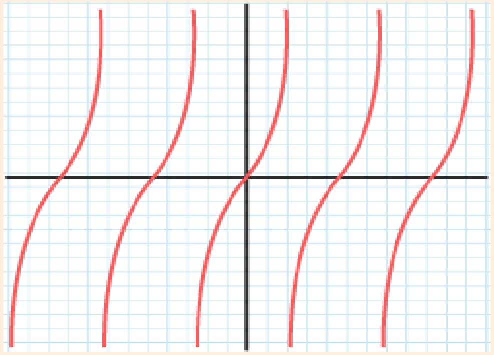
Cos θ



Domain = $(-\infty, +\infty)$

Range = $(-1, +1)$

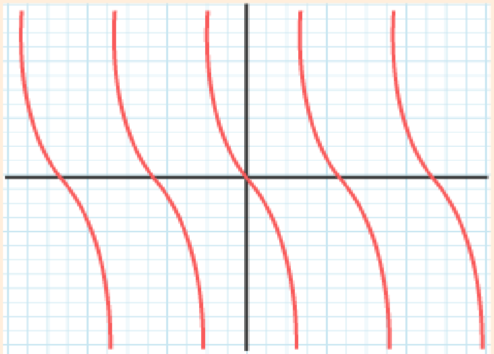
Tan θ



Domain = $R - \frac{(2n+1)\pi}{2}$

Range = $(-\infty, +\infty)$

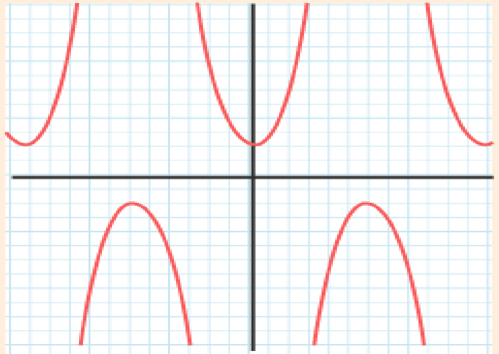
Cot θ



Domain = $(-\infty, +\infty)$

Range = $(-1, +1)$

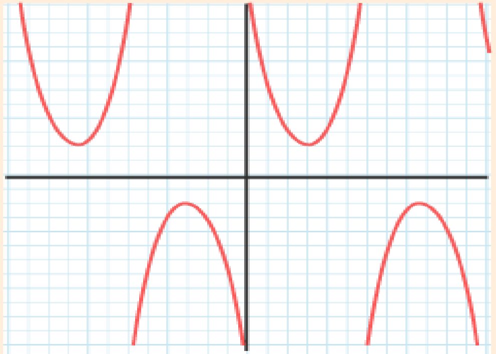
Sec θ



Domain = $R - \frac{(2n+1)\pi}{2}$

Range = $(-\infty, -1][+1, +\infty)$

Cosec θ



Domain = $R - n\pi$

Range = $(-\infty, -1][+1, +\infty)$

Examples on Domain and Range of Trig Functions

Example 1: Find the domain and range of y = sin(x) for all real numbers.

Solution:

The domain and range of y will be as follows:

Domain: R

Range: [−1,1]

Example 2: Determine the domain and range of y = 2cos(3x) for all real values of x.

Solution:

The domain and range of y will be as follows:

Domain: R

Range: [−2,2]

Example 3: Determine the value of y = cot(2x) for x = 8π

Solution:

The value of y at x= 8π

= cot(2× 8π)

= 1

MATH L.O 10

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What is the difference between identities and equations?

IDENTITY : IT is a true equality for two sides, and equal for all values of x or the variable.

For example: $sin(90 - \theta) = cos\,\theta$, it is an identity because if any value of theta is put in the left side, will give the right side, so it is an identity.

Equation : a mathematical statement that shows that two mathematical expressions are equal

For example : $3x + 5 = 15$, this will be true only when x is provided equals 2

TRIGNOMETRIC IDENTITIES

TRIGNOMETRIC IDENTITIES : Trigonometric Identities are the equalities that involve trigonometry functions and holds true for all the values of variables given in the equation.

The identities:

1-sin = $\frac{opp}{Hyp}$ 4-csc= $\frac{1}{sin}$

2-cos = $\frac{Adj}{Hyp}$ 5-sec= $\frac{1}{cos}$

3-tan =$\frac{opp}{Adj}$ 6-cot= $\frac{1}{tan}$

Pythagorean identities

- Pythagorean identities:
- $sin^2\theta + cos^2\theta = 1$
- $1 + tan^2\theta = sec^2\theta$
- $1 + cot^2\theta = csc^2\theta$
- $tan^2\theta = sec^2\theta - 1$
- $csc^2\theta - cot^2\theta = 1$
- $sec^2\theta - tan^2\theta = 1$
- $sec^2\theta - 1 = tan^2\theta$

Cofunction identities

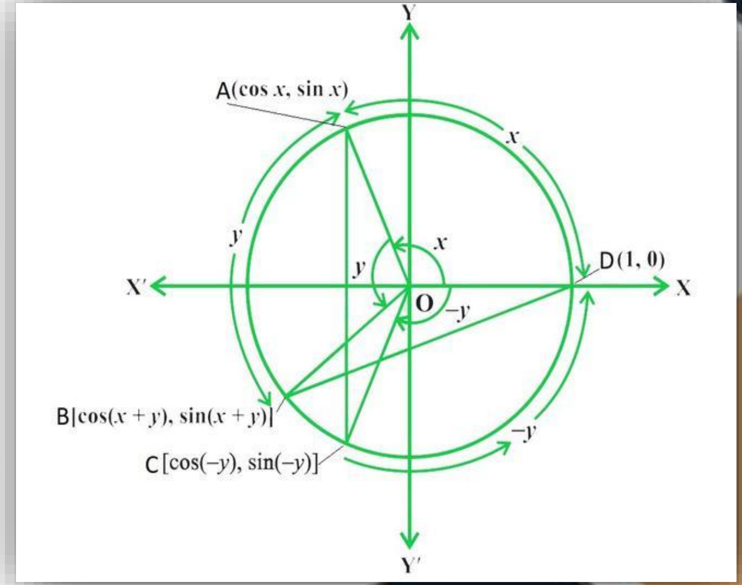
- Cofunction identities:
- $sin(90-\theta) = cos\,\theta$
- $cos(90-\theta) = sin\,\theta$
- $tan(90-\theta) = cot\,\theta$
- $cot(90-\theta) = tan\,\theta$
- $sec(90-\theta) = csc\,\theta$
- $csc(90-\theta) = sec\,\theta$

Trigonometric functions of sum and difference of angles

The following equalities in trigonometry will be used in the upcoming discussion to establish a relation between the sum and difference of angles

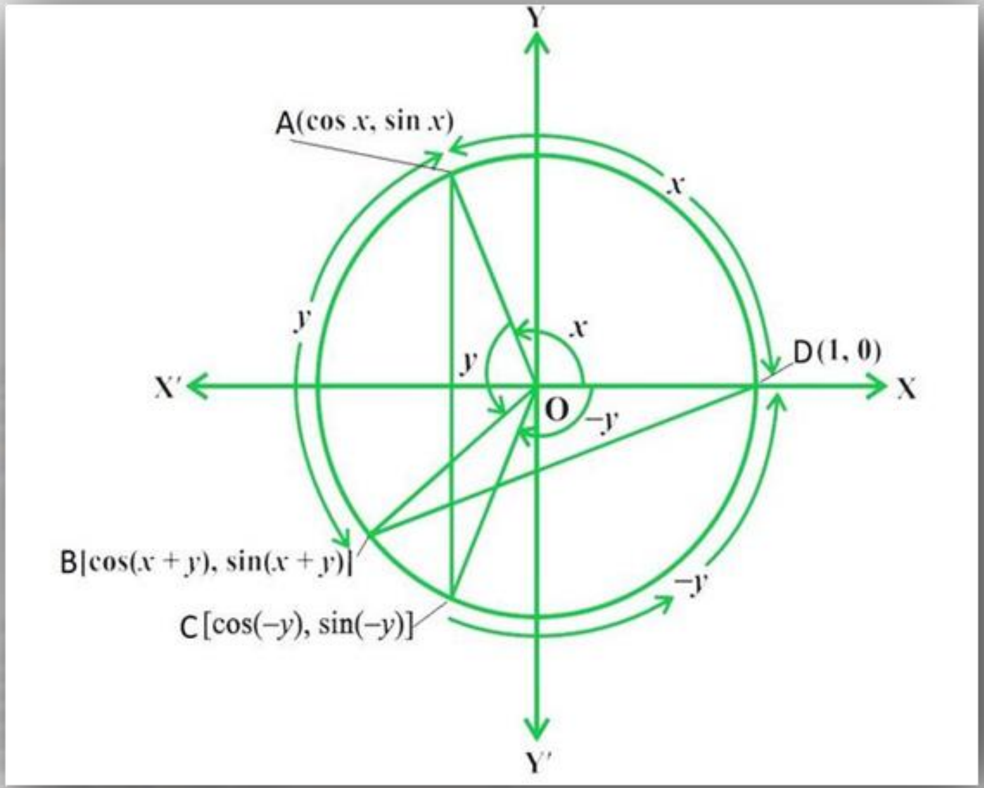
- **cos (-x) = cos x**
- **sin (-x) = -sin x**

Let's consider a unit circle (having radius as 1) with center at the origin. Let x be the ∠DOA and y be the ∠AOB. Then (x + y) is the ∠DOB. Also let (– y) be the ∠DOC.
Therefore, the coordinates of A, B, C and D are



A = (cos x, sin x)
B = [cos (x + y), sin (x + y)]
C = [cos (– y), sin (– y)]
D = (1, 0).

As, ∠AOB = ∠COD
Adding, ∠BOC both side, we get
∠AOB + ∠BOC = ∠COD + ∠BOC
∠AOC = ∠BOD
In ⊿ AOC and ⊿ BOD
OA = OB (radius of circle)
∠AOC = ∠BOD (Proved earlier)
OC = OD (radius of circle)
⊿ AOC ≅ ⊿ BOD by SAS congruency.
By using distance formula, for
AC^2 = [cos x – cos (– y)]^2 + [sin x – sin(–y]^2
AC^2 = 2 – 2 (cos x cos y – sin x sin y) ……………..(i)
And, now
Similarly, using distance formula, we get
BD^2 = [1 – cos (x + y)]^2 + [0 – sin (x + y)]^2
BD^2 = 2 – 2 cos (x + y) ……………..(ii)
As, ⊿ AOC ≅ ⊿ BOD
AC = BD, So AC^2 = BD^2
From eq(i) and eq(ii), we get
2 – 2 (cos x cos y – sin x sin y) = 2 – 2 cos (x + y)
So,
cos (x + y) = cos x cos y – sin x sin y
Take y = -y, we get
cos (x + (-y)) = cos x cos (-y) – sin x sin (-y)
cos (x – y) = cos x cos y + sin x sin y
Now, taking
cos (-(x + y)) = cos ((-x) – y) (cos (-θ) = sin θ)
sin (x – y) = sin x cos y – cos x sin y
take y = -y, we get
sin (x – (-y)) = sin x cos (-y) – cos x sin (-y)
sin (x + y) = sin x cos y + cos x sin y



The derived formulae for trigonometric ratios of compound angles are as follows:

- sin (A + B) = sin A cos B + cos A sin B ………………..(1)
- sin (A – B) = sin A cos B – cos A sin B ………………..(2)
- cos (A + B) = cos A cos B – sin A sin B ..………………(3)
- cos (A – B) = cos A cos B + sin A sin B ………………..(4)

Now if we substitute suitable values in identities (1), (2), (3) and (4), we have the following:

- cos (π/2 + x) = -sin x
- sin (π/2 + x) = cos x
- cos (π± x) = – cos x
- sin (π – x) = sin x
- sin (π + x) = – sin x
- sin (2π – x) = -sin x
- cos (2π – x) = cos x

After having a brief idea about the expansion of sum and difference of angles of sin and cos, the expansion for tan and cot is given by

- **tan (x + y) = (tan x + tan y)/ (1-tan x tan y)**
- **tan (x – y) = (tan x – tan y)/ (1+tan x tan y)**

Similarly;

- **cot (x + y) = (cot x cot y – 1)/(cot y + cot x)**
- **cot (x – y) = (cot x cot y + 1)/(cot y – cot x)**

trig identities of double angles and half angels

Double Angle Formulas

The double angle formulas of sin, cos, and tan are,

- $\sin 2A = 2 \sin A \cos A$ (or) $(2 \tan A) / (1 + \tan^2 A)$
- $\cos 2A = \cos^2 A - \sin^2 A$ (or) $2\cos^2 A - 1$ (or) $1 - 2\sin^2 A$ (or) $(1 - \tan^2 A) / (1 + \tan^2 A)$
- $\tan 2A = (2 \tan A) / (1 - \tan^2 A)$

Double Angle Formulas Derivation

Let us derive the double angle formula(s) of each of sin, cos, and tan one by one.

Double Angle Formulas of Sin

The sum formula of sine function is,

$\sin(A + B) = \sin A \cos B + \cos A \sin B$

When A = B, the above formula becomes,

$\sin(A + A) = \sin A \cos A + \cos A \sin A$

$\sin 2A = 2 \sin A \cos A$

Half Angle Identities

Here are the popular half angle identities that we use in solving many trigonometry problems are as follows:

- Half angle formula of sin: $\sin A/2 = \pm\sqrt{[(1 - \cos A) / 2]}$
- Half angle formula of cos: $\cos A/2 = \pm\sqrt{[(1 + \cos A) / 2]}$
- Half angle formula of tan: $\tan A/2 = \pm\sqrt{[1 - \cos A] / [1 + \cos A]}$ (or) $\sin A / (1 + \cos A)$ (or) $(1 - \cos A) / \sin A$

Half Angle Formula of Sin Proof

Now, we will prove the half angle formula for the sine function.
Using one of the above formulas of cos A, we have

$\cos A = 1 - 2\sin^2(A/2)$

From this,

$2\sin^2(A/2) = 1 - \cos A$

$\sin^2(A/2) = (1 - \cos A) / 2$

$\sin(A/2) = \pm\sqrt{[(1 - \cos A) / 2]}$

Half Angle Formula of Cos Derivation

Now, we will prove the half angle formula for the cosine function. Using one of the above formulas of cos A,

$\cos A = 2\cos^2(A/2) - 1$

From this,

$2\cos^2(A/2) = 1 + \cos A$

$\cos^2(A/2) = (1 + \cos A) / 2$

$\cos(A/2) = \pm\sqrt{[(1 + \cos A) / 2]}$

Half Angle Formula of Tan Derivation

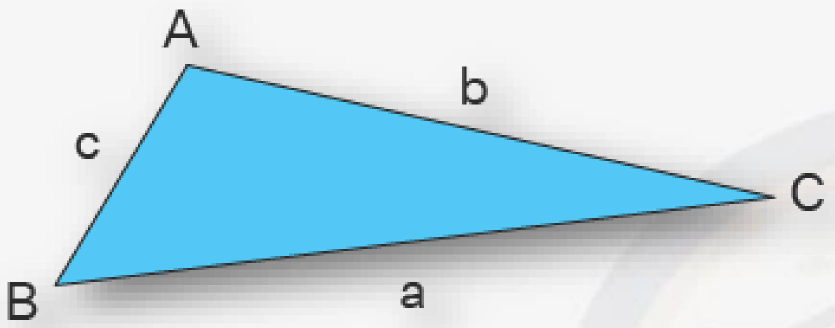
We know that $\tan(A/2) = [\sin(A/2)] / [\cos(A/2)]$

From the half angle formulas of sin and cos,

$\tan(A/2) = [\pm\sqrt{(1 - \cos A)/2}] / [\pm\sqrt{(1 + \cos A)/2}]$

$= \pm\sqrt{[(1 - \cos A) / (1 + \cos A)]}$

Half Angle Formulas Using Semiperimeter



$$S = \frac{a+b+c}{2}$$

$$\sin\left(\frac{A}{2}\right) = \sqrt{\frac{(s-b)(s-c)}{bc}}$$

$$\sin\left(\frac{B}{2}\right) = \sqrt{\frac{(s-c)(s-a)}{ca}}$$

$$\sin\left(\frac{C}{2}\right) = \sqrt{\frac{(s-a)(s-b)}{ab}}$$

$$\cos\left(\frac{A}{2}\right) = \sqrt{\frac{s(s-a)}{bc}}$$

$$\cos\left(\frac{B}{2}\right) = \sqrt{\frac{s(s-b)}{ca}}$$

$$\cos\left(\frac{C}{2}\right) = \sqrt{\frac{s(s-c)}{ab}}$$

$$\tan\left(\frac{A}{2}\right) = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$$

$$\tan\left(\frac{B}{2}\right) = \sqrt{\frac{(s-c)(s-a)}{s(s-b)}}$$

$$\tan\left(\frac{C}{2}\right) = \sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$$

MATH L.O 11

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System of equations

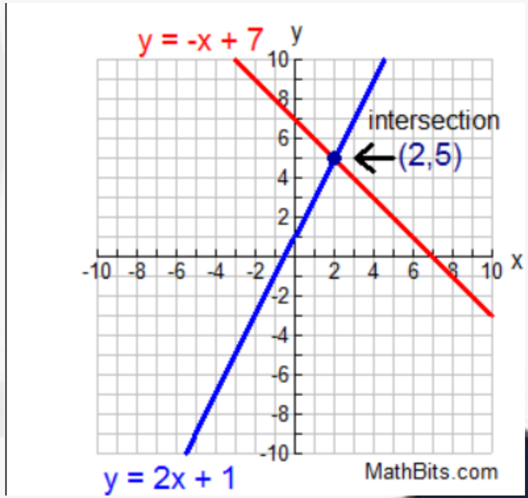
- **A system of equations is a set or collection of equations that are dealt together. These equations can be solved graphically and algebraically. The point where two lines intersect is the solution to the system of equations.**
- Example :
- $3x - 2y = 7$ **equation (1)**
- $2y + 6x = 9$ **equation(2)**
- **By adding the two equations we obtain:**
- $9x = 16$ **then** $x = \frac{16}{9}$
- **By substituting in equation one** : $y = -\frac{5}{6}$

- We can find the solution to a system of equations by graphing the equations:
- 1-Graph the first equation
- 2-Graph the second equation on the same rectangular coordinate system.
- 3-Determine whether the lines intersect, are parallel, or are the same line.
- 4-Identify the solution to the system. If the lines intersect, identify the point of intersection.
- For example:

Equation one is : $y = -x + 7$

Equation two is: $y = 2x + 1$

Since there is point of intersection, then (2,5) can satisfy the equation.



Matrix

- **Matrix is a system of number composed in shape of row and columns as the column is the vertical line of numbers and the horizontal line is the row . In the following matrix: -**
- $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ **:ab and cd are rows and ac and bd are columns so this matrix is called 2 x 2 matrix .**
- **Equal of two matrices: -**

Equal of two matrices needs that the two must have the same order of columns and rows as example if we have two matrices of order 2x2 as the following: - and then the two are equal if $A11 = B11$ and $A21 = B12$ and $B21 = A12$.

Trans pose of matrix: trans pose means converting row to column and column to row.

- Symmetric and skew symmetric matrix :
- Symmetric matrix means that matrix equals its transpose.
- Skew Symmetric matrix means that matrix equals to its negative transpose.

Operations of matrices:

Addition and subtraction: -

For adding or subtracting two matrices they must the same order and then you will add the corresponding numbers.

Multiplying matrices:-

In multiplying two matrices we multiply the first row of first matrix by each column in the second matrix to form the first column in the resultant matrix.

OPERATIONS OF MATRICES:

ADDITION AND SUBTRACTION: -

FOR ADDING OR SUBTRACTING TWO MATRICES THEY MUST BE IN THE SAME ORDER AND THEN YOU WILL ADD THE NUMBERS

IN THE SAME PLACES AS THE FOLLOWING EXAMPLE: -

IF WE HAVE TWO MATRICES 3 4

5 6

AND 1 2

4 3

THEN THE RESULT OF ADDING THEM EQUAL (3 + 1) (4 + 2)

(5 + 4) (6 + 3)

WHICH EQUALS

4 6

9 9

IN THE SECOND MULTIPLYING MATRICES

IN MULTIPLYING TWO MATRICES WE MULTIPLY THE FIRST ROW OF THE FIRST MATRIX BY EACH COLUMN MATRIX TO FORM THE FIRST COLUMN IN THE RESULTANT MATRIX AND SO ON AS THE FOLLOWING EXAMPLE: -

IF A =
3 1 5
2 4 6
AND B =
2 3
0 2
4 5
THEN WHAT IS AB?

[3X2 + 1X0 + 5X4] [3X3 + 2X1 + 5X5]
[2X2 + 4X0 + 6X4] [2X3 + 4X2 + 6X5]

IN MATHEMATICAL OPTIMIZATION, A FEASIBLE REGION,FEASIBLE SET, SEARCH SPACE, OR SOLUTION SPACE IS THE SET OF ALL
POSSIBLE POINTS (SETS OF VALUES OF THE CHOICE VARIABLES) OF AN OPTIMIZATION PROBLEM THAT SATISFY THE PROBLEM'S
CONSTRAINTS, POTENTIALLY INCLUDING INEQUALITIES, EQUALITIES, AND INTEGER CONSTRAINTS.

AN OPTIMAL SOLUTION IS A FEASIBLE SOLUTION WHERE THE OBJECTIVE FUNCTION REACHES ITS MAXIMUM (OR MINIMUM) VALUE – FOR EXAMPLE, THE MOST PROFIT OR THE LEAST COST. A GLOBALLY OPTIMAL SOLUTION IS ONE WHERE THERE ARE NO
OTHER FEASIBLE SOLUTIONS WITH BETTER OBJECTIVE FUNCTION VALUES

MATH L.O 12

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What Are Determinants?

Determinants are considered as a scaling factor of matrices. They can be considered as functions of stretching out and the shrinking in of the matrices. Determinants take a square matrix as the input and return a single number as its output.

For every square matrix, C = [*cij*] of order n×n, a determinant can be defined as a scalar value that is real or a complex number, where *cij* is the (i, j)th element of matrix C. The determinant can be denoted as det(C) or |C|, here the determinant is written by taking the grid of numbers and arranging them inside the absolute value bars instead of using square brackets.

How To Calculate Determinant?

For the simplest square matrix of order 1×1 matrix, which only has only one number, the determinant becomes the number itself. Let's learn how to calculate the determinants for the second order, third order, and fourth-order matrices.

Determinants

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For every square matrix $A = [a_{ij}]$ of order n, we can associate a number called determinant of square matrix. It is denoted by $|A|$ or det(A).

Evaluating Determinants

(1) Order One:

$$A = [\,a\,]$$
$$|A| = |\,a\,| = a$$

(2) Order Two:

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$|A| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$$

(3) Order Three:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$|A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

Properties Of Determinants

(1) Property 1: Interchanging rows with columns

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

(2) Property 2: Interchanging any two rows/ columns

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = - \begin{vmatrix} b_1 & b_2 & b_3 \\ a_1 & a_2 & a_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

(3) Property 3: When any two rows/ columns are equal

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = 0$$

Determinant of a Matrix

The determinant of a 2 x 2 matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is $ad - bc = 0$

The determinant of a 3 x 3 matrix $\begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix}$ is

$$a_1 \cdot \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - a_2 \cdot \begin{vmatrix} b_1 & c_1 \\ b_3 & c_3 \end{vmatrix} + a_3 \cdot \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix} = 0$$

$$a_1(b_2c_3 - b_3c_2) - a_2(b_1c_3 - b_3c_1) + a_3(b_1c_2 - b_2c_1) = 0$$

Properties of Determinants

For square matrices of different types, when its determinant is calculated, they are calculated based on certain important properties of the determinants. Here is the list of some of the important properties of the determinants:

Property1: "The determinant of an identity matrix is always 1“
Property 2: "If any square matrix B with order n×n has a zero row or a zero column, then det(B) = 0“
Property 3: "If C is upper or a lower-triangular matrix, then det(C) is the product of all its diagonal entries“
Property 4: "If D is a square matrix, then if its row is multiplied by a constant k, then the constant can be taken out of the determinant"

Rules For Operations on Determinant

The following rules are helpful to perform the row and column operations on determinants.

- The value of the determinant remains unchanged if the rows and columns are interchanged.
- The sign of the determinant changes, if any two rows or (two columns) are interchanged.
- If any two rows or columns of a matrix are equal, then the value of the determinant is zero.
- If every element of a particular row or column is multiplied by a constant, then the value of the determinant also gets multiplied by the constant.
- If the elements of a row or a column are expressed as a sum of elements, then the determinant can be expressed as a sum of determinants.
- If the elements of a row or column are added or subtracted with the corresponding multiples of elements of another row or column, then the value of the determinant remains unchanged.

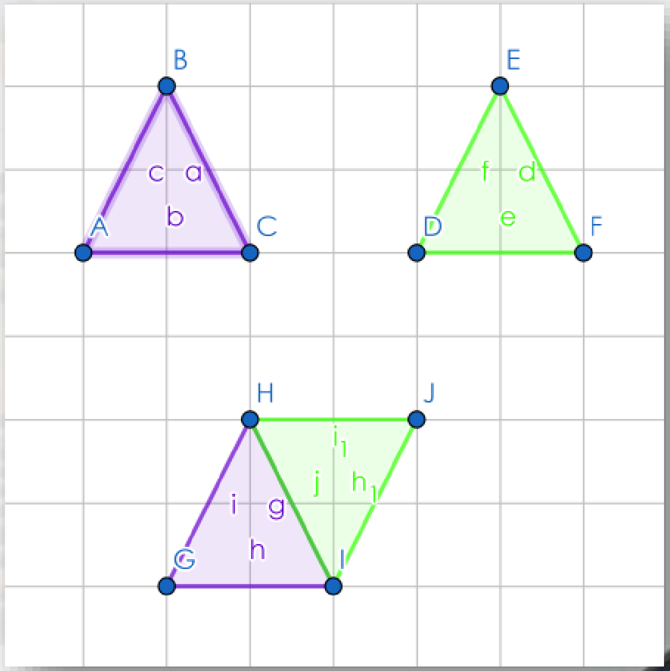
Important Notes on Determinant:

Here is a list of a few points that should be remembered while studying determinant:

- A determinant can be considered as function that takes a square matrix as the input and returns a single number as its output.
- A square matrix can be defined as a matrix that has an equal number of rows and columns.
- For the simplest square matrix of order 1×1 matrix, which only has only one number, the determinant becomes the number itself.

Area ,parallelogram ,and triangle

The area of a triangle is half the length of its base times its height, while the area of a parallelogram is just the length of its base times its height.



How do the areas of triangles and parallelograms relate to each other?

The area of a triangle is half the length of its base times its height, while the area of a parallelogram is just the length of its base times its height. Two congruent triangles make up a parallelogram, and a parallelogram can be evenly divided into two congruent triangles.

What is the difference between an equation and a system of equations?

A linear equation can be graphed by plugging in values for x, and then calculating the corresponding y-values. These x– and y-coordinates can be graphed and will eventually form a line. When there is more than one equation, and they share the same variables, the equations are called a "system".

An example of a system of equations:

y=2x+5$y=2x+5$

3y=4x−8

How are systems of equations used in real life?

Solving systems of equations is a valuable skill to study because of its many applications in the real-world. Anytime we have two unknown values, and enough information to compare the two values, we can solve for the unknowns by setting up a system of equations. Remember, the solution of a system of equations is the value for each variable that makes both of the equations true. This skill has applications in many areas of our daily lives. For example, we can use a system of equations to determine how many calories we burn using different machines at the gym. If we use the rowing machine for 3030 minutes and the treadmill for 2020 minutes, and we burn a total of 430430 calories, this can be set up as equation #1: 30r+20w=430$30r+20w=430$. If we go to the gym the following day and use the rowing machine for 5050 minutes and the treadmill for 1010 minutes, burning a total of 600600 calories, this can be set up as equation #2: 50r+10w=600$50r+10w=600$. We have two unknown values (how many calories we burn per minute on each machine), and we have enough information comparing both of the values. This means that we can solve the problem by setting up and solving a system of equations.

How do you solve systems of equations by substitution?

Solving systems of equations by substitution follows three basic steps.

<u>Step 1:</u> Solve one equation for one of the variables.

<u>Step 2:</u> Substitute this expression into the other equation, and solve for the missing variable.

<u>Step 3:</u> Substitute this answer into one of the equations in order to solve for the other variable.

What is Cramer's Rule?

Cramer's rule is one of the methods used to solve a system of equations. This rule involves determinant. i.e., the values of the variables in the system are found with the help of determinants. Let us consider a system of equations in n variables x_1, x_2, x_3, ..., x written in the matrix form AX = B, where

- A = the coefficient matrix which is a square matrix
- X = the column matrix with variables
- B = the column matrix with the constants (which are on the right side of the equations)

Cramer's Rule

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To find the solution of the system $Ax = B$

- Find the following determinants.

$$D = |A|, Dx_1, Dx_2, \ldots, Dx_n$$

Where Dx_i is the same determinant as D where i^{th} column is replaced with B.

- Apply,

$$X_1 = \frac{Dx_1}{D} \; ; \; X_2 = \frac{Dx_2}{D} \; ; \ldots\ldots ; \; X_n = \frac{Dx_1}{D}$$

(Where $D \neq 0$)

Cramer's Rule For 2 x 2

Using the above formula, let us see how to solve a system of 2 equations in 2 variables using Cramer's rule. Here are the steps to solve this system of 2x2 equations in two unknowns x and y using Cramer's rule.

- **Step-1:** Write this system in matrix form is AX = B.
- **Step-2:** Find D which is the determinant of A. Also, find the determinants D_x and D_γ where

D_x = det (A) where the first column is replaced with B

D_γ = det (A) where the second column is replaced with B

- **Step-3:** Find the values of the variables x and y by dividing each of D_x and D_γ by D respectively.

Cramer's Rule For 3 x 3

We will just extend the same process of Cramer's rule for 2 equations for a 3x3 system of equations as well. Here are the steps to solve this system of 3x3 equations in three variables x, y, and z by applying Cramer's rule.

- **Step-1:** Write this system in matrix form is AX = B.
- **Step-2:** Find D which is the determinant of A. i.e., D = det (A). Also, find the determinants D_x, D_γ, and D_z where

D_x = det (A) where the first column is replaced with B

D_γ = det (A) where the second column is replaced with B

D_z = det (A) where the third column is replaced with B

- **Step-3:** Find the values of the variables x, y, and z by dividing each of D_x, D_γ, and D_z by D respectively.

Cramer's Rule Chart

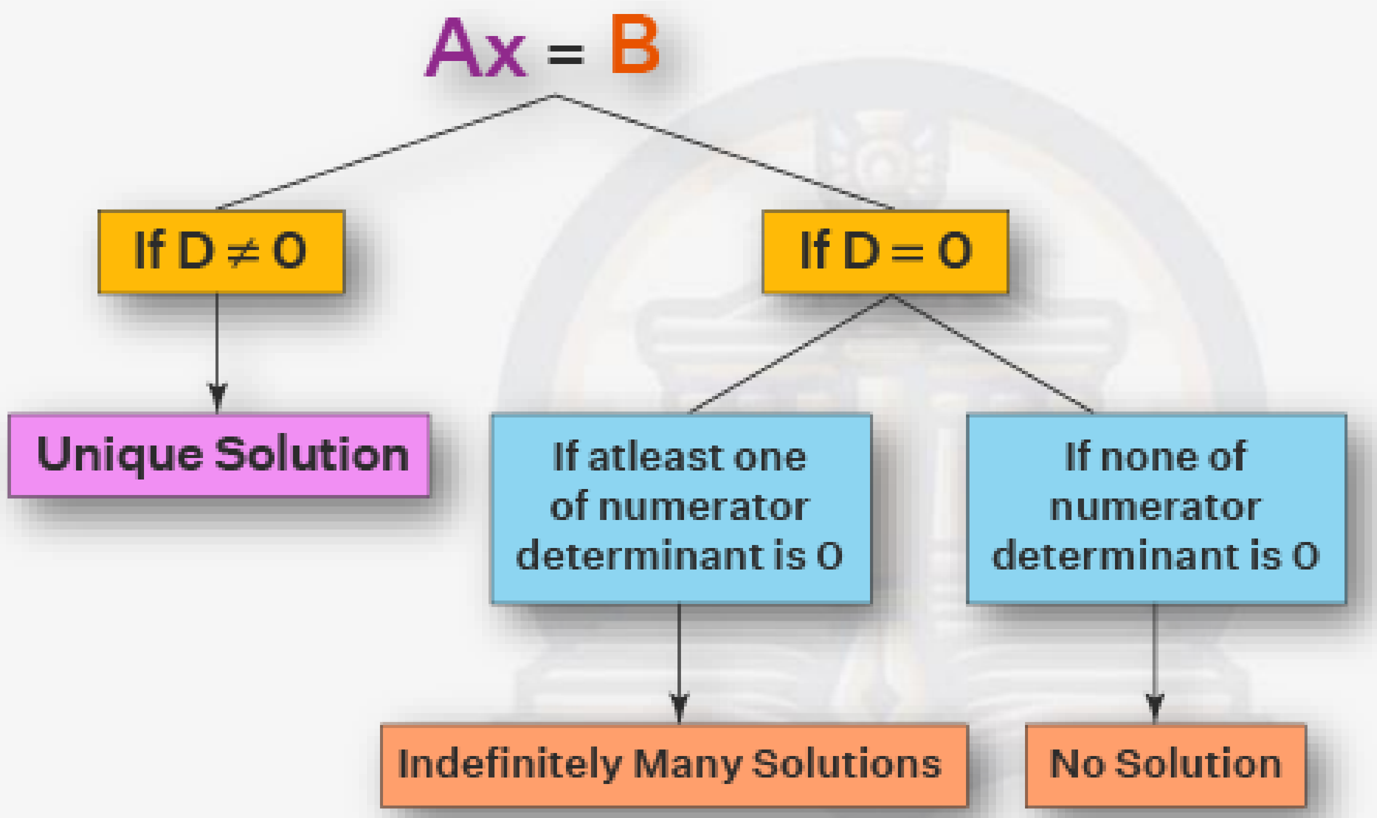
If we observe the formula of Cramer's rule in all the above three sections, we have mentioned that $D \neq 0$ everywhere. This is because while finding the values of the variables, D is in the denominator and if $D = 0$, the fraction (the value of the variable) goes undefined. So this rule is applicable only when $D \neq 0$. But what about the system of equations when $D = 0$? Then there are two possibilities.

- **The system may have no solution.**
- **The system may have an infinite number of solutions.**

Though Cramer's rule doesn't help in finding the infinite number of solutions, we can determine whether the system has "no solution" or "infinite number of solutions" using the determinants which we compute as the process of applying the rule.

- **If $D \neq 0$, we say that the system $AX = B$ has unique solution.**
- **If $D = 0$ and at least one of the numerator determinants is a 0, then the system has infinitely many solutions.**
- **If $D = 0$ and none of the numerator determinants is 0, then the system has no solution.**

Cramer's Rule Chart



Cramer's Rule Condition

From the above chart and explanation, it is very clear that Cramer's rule is NOT applicable when D = 0. i.e., when the determinant of the coefficient matrix is 0, we cannot find the solution of the system of equations using Cramer's rule. In this case, we can find the solution (if any) by using Gauss Jordan Method.

Thus, Cramer's rule is used to find the solution of a system only when the system has a unique solution.

Important Notes on Cramer's Rule:

Here are some important notes related to the application of Cramer's rule:

- If there are n variables and n equations, we have to compute (n + 1) determinants.
- This rule can give the solutions only when $D \neq 0$.
- If $D = 0$, the system has either an infinite number of solutions or no solutions.
- We cannot find solutions by using this rule when the system has an infinite number of solutions.

THANK YOU

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